Corrosion of steel reinforcing bars is the most important durability problem for reinforcing structures. Carbonation of concrete results from a chemical reaction that lowers the pH value by enough to initiate corrosion of the rebar. Data on the carbonation depth (mm) and strength (MPa) for a sample of core specimens was taken from a particular building, and all the regression output is provided. We are interested in modeling the strength (from carbonation).

x=carbonation, y=strength

Scatterplot: relatively strong, negative, linear relationship.

Population model:

$$y = \beta_0 + \beta_1 x + \epsilon$$

y= response variable
β_0= intercept (value of y when x=0)
β_1=slope (change in y due to one unit increase in x)
x=explanatory (dependent) variable
ϵ= residual term

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Coefficients:
            Estimate Std. Error t value Pr(>|t|)
(Intercept) 27.18294    1.65135  16.461 1.88e-11 ***
carbonation -0.29756    0.04116  -7.229 2.01e-06 ***
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Signif. codes:  0 ‘***’ 0.001 ‘**’ 0.01 ‘*’ 0.05 ‘.’ 0.1 ‘ ’ 1

Residual standard error: 2.864 on 16 degrees of freedom
Multiple R-squared:  0.7656,	Adjusted R-squared:  0.7509
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Equation with numerical coefficients:

$$\hat{y} = 27.18 - 0.298x$$

Using the equation, estimate the strength when carbonation depth is 8mm and again for 20mm. The observed values of strength for those x-values are, respectively, 22.8 and 17.1 MPa

(8,22.8), (20,17.1)

Estimates with sample model equation:

$$\hat{y}_{|x=8} = 27.18 - 0.298(8) = 24.8\ MPa$$

$$\hat{y}_{|x=20} = 27.18 - 0.298(20) = 21.2\ MPa$$

Residuals of sample model estimates:

Residual: $e = y - \hat{y}$

$$e_{|x=8} = 22.8 - 24.8 = -2 < 0$$

$$e_{|x=20} = 17.1 - 21.2 = -4.1 < 0$$

Model has overestimated in these two cases since the residuals are < 0

Interpretation of slope and intercept in context:

Slope ($\hat{\beta}_1$): when carbonation depth increases by one unit (1mm), strength of concrete decreases by 0.298

Intercept ($\hat{\beta}_0$): when carbonation depth is 0 (x=0), strength (y) is 27.18. While there is no x=0 in the dataset, this could make logical sense.

Hypotheses for slope test (no intercept test because I am not sure it is appropriate since there is no x=0 in dataset but intercept could (?) make logical sense in context)

$$H_0: \beta_1 = 0\ vs. H_a: \beta_1 \neq 0$$

$t = -7.229, pvalue = 2.01e - 06 = 2.01 \times 10^{-6} \approx 0 \leq \alpha(0.05) \therefore H_0$ is rejected. The slope is significant (the relationship between x and y is significant)

$R^2 = 0.7656 = 76.56\%$ of the variation in the response can be explained by the linear model (relationship between strength and carbonation). $R^2 = 0.7656 = 76.56\% \geq 60\%$, which is good.

$r = \pm\sqrt{R^2} = -\sqrt{0.7656} = -0.87$, since $r > |0.8|$ this is a decently strong, negative, linear relationship

Assumptions:
1. Mean of residuals approx. 0
2. Variance of residuals is constant (same for all values)
3. Independence of residuals
4. Normality of residuals

Assessment of model: slope is significant, both r and R^2 are good, this is a good model